# Adaptive Experimental Designs

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# Agenda

# Overview of Experiments

- A randomized experiment is the gold standard for making causal inferences
- Randomization of the treatment will make the treatment and control groups similar on average with respect to *observed* and *unobserved* covariates
- **Advantage 1**: Identification is justified by design of experiments
  - ⋆ We control the treatment assignment mechanism
  - ⋆ We do not need to make "ignorability"-type assumptions
- **Advantage 2**: Estimation is simple
  - ⋆ Difference-in-means (DiM) or some weighted averages of DiM
- **Advantage 3**: Inference is simple
  - ⋆ We can again use the known treatment assignment mechanism as a "reason basis for inference"

# Basic Setup for Randomized Experiment

- Units: $i \in \{1, \ldots, N\}$
- Treatment: $T_i \in \{0, 1\}$, randomly assigned
- Potential outcomes: $Y_i(0)$ and $Y_i(1)$
- Observed outcome: $Y_i = T_i Y_i(1) + (1 - T_i) Y_i(0)$ (consistency)

- Treatment Assignment Mechanism

  (1) Complete randomization: Exactly $N_1$ units are treated

  (2) Bernoulli (simple) randomization: Each unit is independently assigned to treatment with probability $p$

- Randomization (complete or simple) implies

$$\{Y_i(1), Y_i(0)\} \perp\!\!\!\perp T_i$$

# Identification of Average Treatment Effect

- Causal Estimand: Average Treatment Effect

$$\tau_{ATE} \equiv \mathbb{E}\{Y_i(1) - Y_i(0)\}$$

- Not directly estimable as we don't observe $Y_i(1) - Y_i(0)$ for each unit
- Identification Question:
  Can we write down $\tau_{ATE}$ only with observed data $(Y_i, T_i)$?

$$\begin{aligned} & \mathbb{E}\{Y_i(1) - Y_i(0)\} \\ = \;& \mathbb{E}\{Y_i(1)\} - \mathbb{E}\{Y_i(0)\} \quad (\because \text{Linearity of } \mathbb{E}) \\ = \;& \mathbb{E}\{Y_i(1) \mid T_i = 1\} - \mathbb{E}\{Y_i(0) \mid T_i = 0\} \quad (\because \text{Randomization of } T) \\ = \;& \mathbb{E}(Y_i \mid T_i = 1) - \mathbb{E}(Y_i \mid T_i = 0) \quad (\because \text{Consistency of PO}) \end{aligned}$$

- Estimation Question: Can we estimate $\mathbb{E}(Y_i \mid T_i = 1) - \mathbb{E}(Y_i \mid T_i = 0)$?

$$\frac{1}{N_1}\sum_{i=1}^{N} T_i Y_i - \frac{1}{N_0}\sum_{i=1}^{N}(1 - T_i) Y_i$$

# Considering Many Different Treatments

- What if we want to learn about the effects of many separate treatments?
- **Multi-arm trials**: instead of testing 1 treatment, test $K$ treatments
- Units: $i \in \{1, \ldots, N\}$
- Treatments: $T_i \in \{1, 2, \ldots, K\}$, randomly assigned
- Potential outcomes: $Y_i(1)$, $Y_i(2)$, $\ldots$, $Y_i(K)$
- Observed outcome: $Y_i = \sum_{k=1}^{K} \mathbb{1}[T_i = k] Y_i(k)$

# Identification of Average Treatment Effects

- Here, we can make pairwise comparisons between arms:

$$\tau_{ATE} \equiv \mathbb{E}\{Y_i(k) - Y_i(k')\}$$

- Where $k$ and $k'$ are two separate treatment groups
- Not directly estimable as we don't observe $Y_i(k) - Y_i(k')$ for each unit
- Identification: can we write down $\tau_{ATE}$ only with observed data $(Y_i, T_i)$?

$$\begin{aligned}
& \mathbb{E}\{Y_i(k) - Y_i(k')\} \\
= \; & \mathbb{E}\{Y_i(k)\} - \mathbb{E}\{Y_i(k')\} \quad (\because \text{ Linearity of } \mathbb{E}) \\
= \; & \mathbb{E}\{Y_i(k) \mid T_i = k\} - \mathbb{E}\{Y_i(k) \mid T_i = k'\} \quad (\because \text{ Randomization of } T) \\
= \; & \mathbb{E}(Y_i \mid T_i = k) - \mathbb{E}(Y_i \mid T_i = k') \quad (\because \text{ Consistency of PO})
\end{aligned}$$

- Estimation: can we estimate $\mathbb{E}(Y_i \mid T_i = k) - \mathbb{E}(Y_i \mid T_i = k')$?

$$\frac{1}{N_k} \sum_{i=1}^{N_k} \mathbb{1}[T_i = k] Y_i(k) - \frac{1}{N_{k'}} \sum_{i=1}^{N_{k'}} \mathbb{1}[T_i = k'] Y_i(k')$$

- Notice that this is an analogous generalization of the simple experiment!

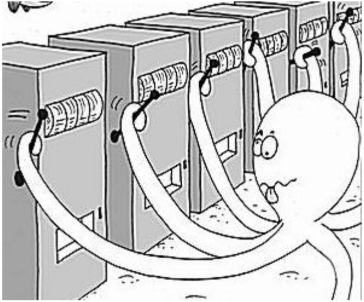
# Motivating Adaptive Designs

If we're interested in many treatments, we can create several treatment groups:

- Identification is justified by design (randomization)
- Estimation and inference are simple for pair-wise comparisons across arms

So why not just divide our $N$ subjects into $K$ treatment groups?

- Trade-off between exploration and exploitation
- Illustration: the multi-armed bandit



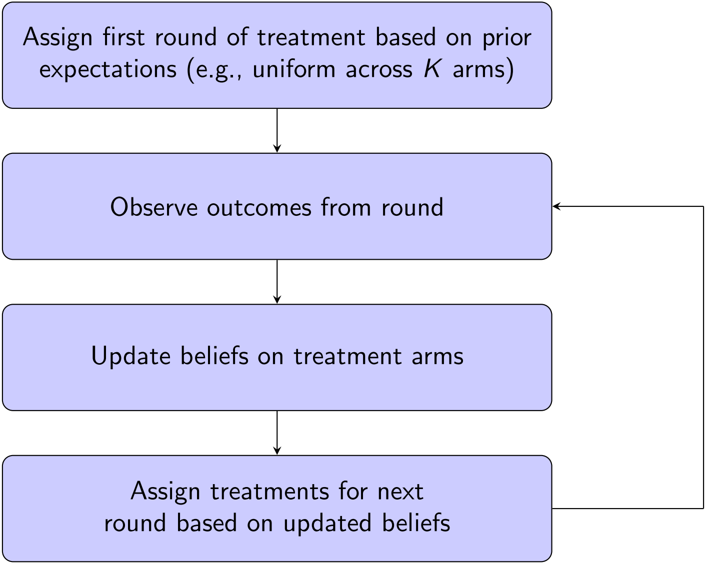
# Motivating Adaptive Designs: Examples

- Get-out-the-vote: normatively, may want to increase turnout
  - Test many interventions, but maximize subjects that get best arm
  - Ethics: maybe this will help you convince an implementing partner to do an RCT (participant welfare maximized)
- Business: company wants to experiment with ads, but maximize click-rate
  - Useful to discard bad ads over time
- Medical: test different drugs, ensure most participants healed as possible
  - Once you know a drug doesn't work, stop giving it to participants
- Other social science applications? Open questions!

Can narrow number of treatments with theory, but what if we miss something?

- Let the data decide which arms are most promising
- Key idea: adapt our treatment assignments based on the outcomes over time
- Increase the speed and efficiency of finding best arm

# Adaptive Designs: General Workflow



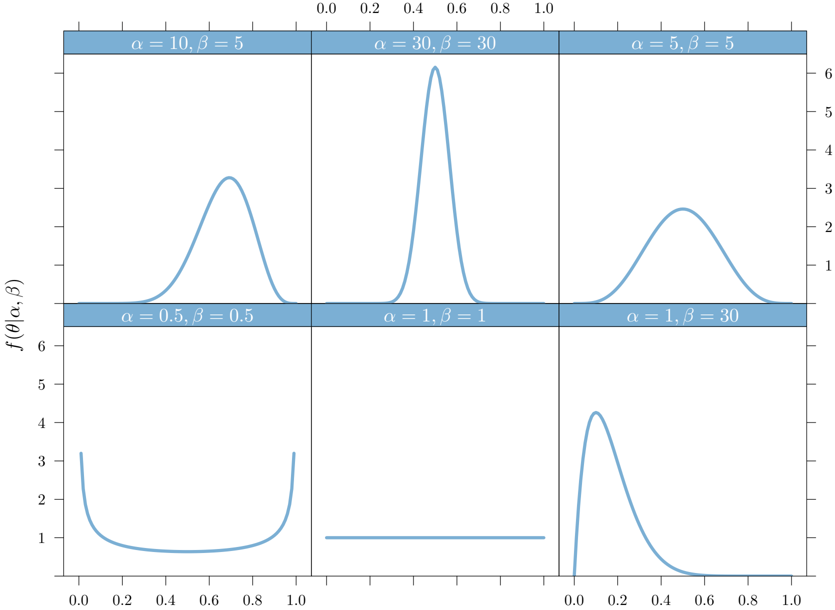
# Setup: Adaptive Designs

- Units: $i \in \{1, \ldots, N\}$
- Treatments: $T_i \in \{1, 2, \ldots, K\}$, randomly assigned
- Potential outcomes: $Y_i(1)$, $Y_i(2)$, $\ldots$, $Y_i(K)$
- Observed outcome: $Y_i = \sum_{k=1}^{K} \mathbb{1}[T_i = k] Y_i(k)$
- New! Time periods (waves): $w$
- For ease of exposition, consider binary outcomes $Y \in \{0, 1\}$

$K$ arms have success rates $\theta_1, \theta_2, \ldots, \theta_k$, where $\theta_k = \Pr(Y = 1 | T = k)$

- Likelihood: $Y_1, \ldots, Y_{n,k} \mid \theta_k \overset{\text{i.i.d.}}{\sim} \text{Bern}_{n,k}(\theta_k)$, where $n, k$ is the number of units in arm $k$
- Can set priors to the success rate of each arm $\theta_k \sim \mathrm{Beta}(\alpha_0^k, \beta_0^k)$
- Let's set agnostic priors: for all $k$, $\theta_k \sim \mathrm{Beta}(1, 1)$

# Recalling the Beta Distribution



# Assigning Treatment Probabilities

In each period $w$, treatment assigned by the probability of each arm being best

- In the first period (wave 0), for agnostic priors $\theta_k \sim \mathrm{Beta}(1, 1)$, this means equal probability of assignment across all arms $k$
- For later periods, we can calculate based on observed outcomes
  - For example, which arm had the maximum outcome for each draw of the posterior distribution divided by total number of draws
  - Depends on what you want to maximize, so define carefully
  - For more, see Kasy, Maximilian, and Anja Sautmann. "Adaptive treatment assignment in experiments for policy choice." *Econometrica* 89.1 (2021): 113-132.
- At the end of each period, the posterior is updated
- $\implies$ Probability that each arm is best is recalculated

# Updating on Best Treatment

Each period $w$, treatment is assigned, observations recorded for each arm $k$

- Cumulative assignment to arm $k$: $n_{k,t}$
- Responses under $k$ in wave $w$

Distribution of each $\theta_k$ given data $Y_k^{\{n_{k,t}\}}$ is then:

$$\underbrace{f\left(\theta_k | Y_k^{\{n_{k,t}\}}\right)}_{\text{Posterior}} \propto \underbrace{p\left(Y_k^{\{n_{k,t}\}} | \theta_k\right)}_{\text{Likelihood}} \cdot \underbrace{f\left(\theta_k\right)}_{\text{Prior}}$$

- Beta distribution is conjugate for binomial likelihoods
- So posteriors follow beta distribution—we can just count!
- Posterior distribution: $f\left(\theta_k | Y_k^{\{n_{k,t}\}}\right) \sim \mathrm{Beta}(\alpha_w^k, \beta_w^k)$
- $\alpha_w^k$ is $\alpha_0^k$ + total successes (1) observed from that arm
- $\beta_w^k$ is $\beta_0^k$ + total failures (0) observed from that arm

# Practical Considerations

- Weight by inverse treatment assignment probabilities for estimation
- Large burn-in period could be useful to ensure results are not driven by noise
- Stop after a fixed number of periods or number of subjects have participated
- If interested in control comparison, useful to do a control-augmented design
  - e.g., Offer-Westort, Molly, Alexander Coppock, and Donald P. Green. "Adaptive experimental design: Prospects and applications in political science." *American Journal of Political Science* 65.4 (2021): 826-844.
  - At each step, parity between best performing condition and control
- In practice, fit a model using MCMC and generate the posterior in that way
- Or use quasi-Bayesian approach: generate "posteriors" based on estimates